

The Greek letters phi ( $\phi$ ) and psi ( $\psi$ ) are used to represent any predicates whatever.

### Rules of Inference: Quantification

Universal Instantiation U.I.  $(x) (\phi x)$

$\therefore \phi v$

(where  $v$  is any individual symbol)

Universal Generalization U.G.  $\phi y$

$\therefore (x) (\phi x)$

(where  $y$  denotes 'any arbitrarily selected individual')

Existential Instantiation E.I.  $(\exists x) (\phi x)$

$\therefore \phi v$

Existential Generalization E.G.  $\phi v$

$\therefore (\exists x) (\phi x)$

(1848-1925),  
Gottlob Frege, one of the  
founders of modern symbolic logic.

~~1848-1925~~ 18

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U.I Any substitution instance  
of a prop. function <sup>can be</sup> ~~one may~~ validly  
inferred <sup>from its</sup> ~~the~~ universal quantification.

U.G. From the substitu<sup>tion</sup> instance of a prop.  
function, one may validly infer the  
Univer<sup>sal</sup> quantification of that prop. function

E.I From the existential quantifi-  
cation of a prop. function, we may infer  
the truth of its substitution instance  
(other than  $y$ ) that occur ~~no~~  
nowhere ~~in the~~ earlier in the context

E.G. From an instance substitution  
of a propo<sup>sitional</sup> function, we may validly  
infer the existential quantification  
of that propositional function.

In the geometer's proof, the only assumption made about  $ABC$  is that it is a triangle; hence what is proved true of  $ABC$  is proved true of any triangle. In our proof, the only assumption made about  $y$  is that it is an individual; hence what is proved true of ' $y$ ' is proved true of any individual. The symbol ' $y$ ' is an individual symbol, but it is a very special one. Typically it is introduced into a proof by using U.I., and only the presence of  $y$  permits the use of U.G.

Here is a valid argument, the demonstration of whose validity requires the use of U.G. as well as U.I. :

No humans are perfect

All Indians are humans.

∴ No Indians are perfect.

The formal proof of its validity is :

$$1. (x) (Hx \supset \sim Px)$$

$$2. (x) (Ix \supset Hx)$$

$$\therefore (x) (Ix \supset \sim Px)$$

$$3. Hy \supset \sim Py \quad 1, U.I.$$

$$4. Iy \supset Hy \quad 2, U.I.$$

$$5. Iy \supset \sim Py \quad 4, 3, H.S.$$

$$6. (x) (Ix \supset \sim Px) \quad 5, U.G.$$

$$1. (x) (Cx \supset Vx)$$

$$2. (\exists x) (Hx \cdot Cx)$$

$$\therefore (\exists x) (Hx \cdot Vx)$$

$$3. Ha \cdot Ca \quad 2, E.I.$$

$$4. Ca \supset Va \quad 1, U.I.$$

$$5. Ca \cdot Ha \quad 3, Com.$$

$$6. Ca \quad 5, Simp.$$

$$7. Va \quad 4, 6, M.P.$$

$$8. Ha \quad 3, Simp.$$

$$9. Ha \cdot Va \quad 8, 7, Conj.$$

$$10. (\exists x) (Hx \cdot Vx) \quad 9, E.G.$$

If we failed to heed the restriction on E.I. from that a substitution instance of a propositional function inferred by E.I. from the existential quantification of that propositional function can contain only an individual symbol (other than  $y$ ) that has no previous occurrence in the ~~context~~ context, then we might proceed to construct a "proof" of validity for the following invalid argument. So an erroneous "proof" might proceed as follows:

$$1. (\exists x) (Ax \cdot Cx)$$

$$2. (\exists x) (Bx \cdot Cx)$$

$$\therefore (\exists x) (Ax \cdot Bx)$$

$$3. Aa \cdot Ca \quad 1, \text{E.I.}$$

$$4. Ba \cdot Ca \quad 2, \text{E.I. (wrong)}$$

$$5. Aa \quad 3, \text{Simp.}$$

$$6. Ba \quad 4, \text{Simp.}$$

$$7. Aa \cdot Ba \quad 5, 6, \text{Conj.}$$

$$8. (\exists x) (Ax \cdot Bx) \quad 7, \text{E.G.}$$

The need for the indicated restriction on the use of E.I. can be seen by considering <sup>the</sup> obviously invalid ~~an~~ argument.