Indirect Proof of Validity

Contradictory statements cannot both be true. Therefore, a statement added to the premises that makes it possible to deduce a contradiction must entail a falsehood. This gives rise to another method of proving validity. Suppose we assume (for the purposes of the proof only) the denial of what is to be proved.

Suppose, using that assumption, we can derive a contradiction. That contradiction will show that when we denied what was to be proved we were brought to absurdity. We will have established the desired conclusion ~~ind~~ indirectly, with a proof ~~by~~ by reductio ad absurdum.

An indirect proof of validity is written out by stating as an additional ~~assumed premise~~ assumed premise the negation of the conclusion.

If we can derive an explicit contradiction from the set of premises thus augmented, the argument with which we began must be valid. The method is illustrated with the following argument:

1. $A \supset (B \cdot C)$
2. $(B \vee D) \supset E$
3. $D \vee A$

$\therefore E$

In the very next line we make explicit our assumption (for the purpose of the indirect proof) of the denial of the conclusion:

4. $\sim E$ I.P. (Indirect Proof)

With the now enlarged set of premises we can, using the established rules of inference, bring out an explicit contradiction thus:

5. $\sim(B \vee D)$ 2, 4, M.T.
6. $\sim B \cdot \sim D$ 5, De. M
7. $\sim D \cdot \sim B$ 6, Com.
8. $\sim D$ 7, Simp.
9. A 3, 8, D.S.
10. $B \cdot C$ 1, 9 M.P.
11. B 10, Simp.
12. $\sim B$ 6, Simp.
13. $B \cdot \sim B$ 11, 12, Conj.