Quantification : Proving Invalidity

The procedure for proving the invalidity of an argument containing general propositions is the following:

1st, consider a one-element model containing only the individual a.

Then write out the logically equivalent truth functional argument for that model, which is obtained by moving from each general proposition (quantified propositional function) of the original argument to the substitution instance of that propositional function with respect to 'a'. If the truth-functional argument can be proved invalid by assigning truth values to its component simple statements, that suffices to prove the original argument invalid.

If that cannot be done, next consider a two-element model containing the individuals a and b.

Where the original argument contains a universally quantified propositional function, $(x)(\phi x)$, the new substitution instance ϕb is combined with the first substitution instance ϕa by conjunction (".") ϕ.

But where the original argument contains an existentially quantified propositional function, $(\exists x)(\phi x)$, the new substitution instance ϕb is combined with the first substitution instance ϕa by disjunction ("v").

If the new truth functional argument can be proved invalid by assigning truth values to its component simple statements, that suffices to prove the original argument invalid

If that cannot be done, next consider a <u>three-element model</u> containing the individuals a, b, and c.

$(x)\ (Cx \supset Ax)$

$(\exists x)\ (Cx \cdot Wx)$

$\therefore\ (x)\ (Wx \supset Ax)$

For a model containing exactly one individual, a, it is logically equivalent to

$Ca \supset Aa$

$Ca \cdot Wa$

$\therefore Wa \supset Aa$

Which is valid. But for a model containing two individuals, a and b, it is logically equivalent to

$(Ca \supset Aa) \cdot (Cb \supset Ab)$

$(Ca \cdot Wa) \vee (Cb \cdot Wb)$

$\therefore (Wa \supset Aa) \cdot (Wb \supset Ab)$

which is proved invalid by assigning true to Ca, Aa, Wa, Wb and false to Cb and Ab.

Hence the original argument is not valid for a model containing exactly two individuals, and it is therefore invalid.